

The Decay of Nearly Flat Space

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[hep-th/0603105](https://arxiv.org/abs/hep-th/0603105)

Motivation

Landscape

Many Vacua

Probability of each vacuum \propto hard

Eternal Inflation

Semi-classical

Large Λ dS dominates?

Ergodic Evolution (Banks & Johnson hep-th/0512141)

$\Lambda_{\min} > 0$ “true” ground, all others are fluctuations

Probability \sim Lifetime \sim Entropy

$\Gamma \propto 0$ for $\Lambda \propto 0$ to stabilize Λ_{\min} dS

but, $\Gamma \neq 0$ (discontinuous) at $\Lambda = 0$

What we did

Investigate CdL equations

- Consider singular “solutions”
- General properties
- Map “solution” space

Γ continuous as $\Lambda \nearrow 0$

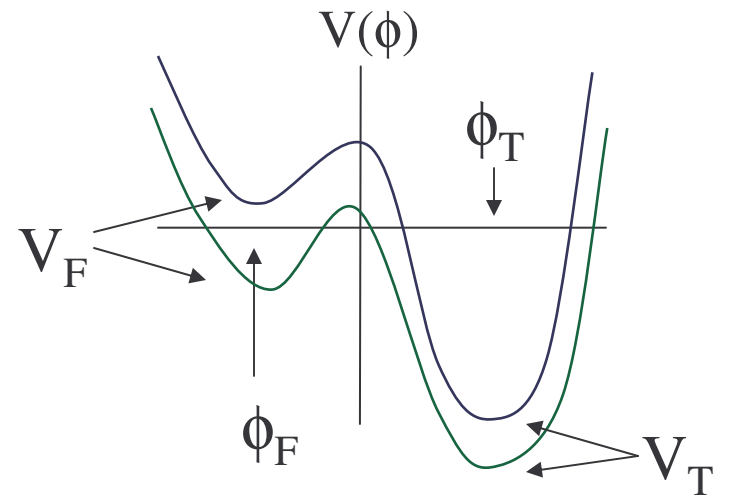
➔ If $\Gamma \nearrow 0$, $\Lambda = 0$ limit is stable

(See also Banks, Johnson, & Aguirre hep-th/0603107)

CdL Tunneling Review

Scalar coupled to gravity

$$S = \int d^4x \sqrt{g} \left(\frac{-M_p^2}{16\pi} R + \frac{1}{2} (\nabla\phi)^2 + V(\phi) \right)$$



Euclidean instanton

$$\Gamma \sim \exp(-S_I + S_{BG})$$

SO(4) symmetry

$$\text{metric: } ds^2 = dt^2 + \rho^2(t) d\Omega_3^2$$

S^3

Lorentzian dynamics

expanding bubble of true vacuum

$$\left\{ \begin{array}{l} V_T > 0 \quad \text{dS} \\ V_T = 0 \quad \text{open FRW} \\ V_T < 0 \quad \text{big crunch} \end{array} \right.$$

Equations of Motion

$$\ddot{\phi} + 3\frac{\dot{\rho}}{\rho}\dot{\phi} = V'(\phi) \quad \longrightarrow \quad \text{Particle}$$

in potential $-V(\phi)$
with friction $\sim H = \frac{\dot{\rho}}{\rho}$

$$\ddot{\rho} = -\frac{4\pi}{3M_p^2}\rho\left(\dot{\phi}^2 + V(\phi)\right)$$

$$\dot{\rho}^2 - 1 = \frac{8\pi}{3M_p^2}\rho^2\left(\frac{\dot{\phi}^2}{2} - V(\phi)\right)$$

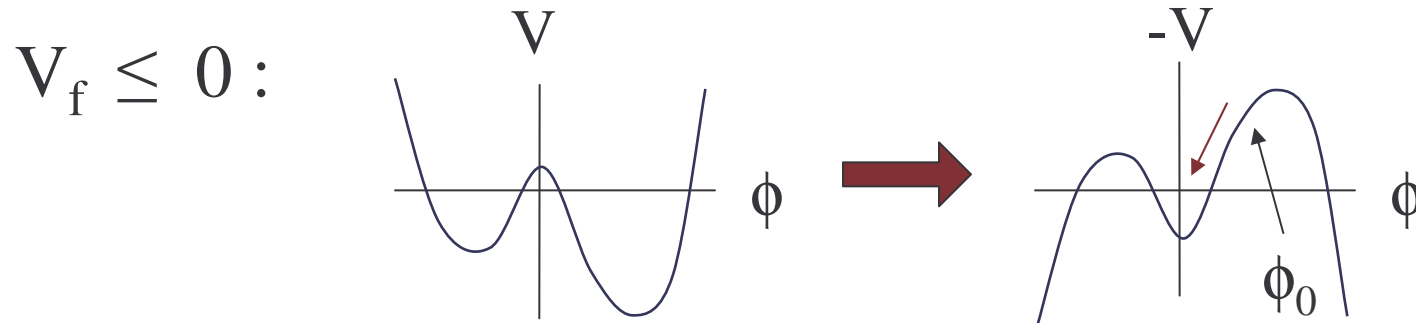
Coupled to FRW

Boundary Conditions
at $\rho = 0$ poles

$$\dot{\phi} = 0 \quad \blacktriangle \text{ Continuous}$$

$$\dot{\rho}^2 = 1 \quad \blacktriangle \text{ Smooth}$$

Solutions - Noncompact



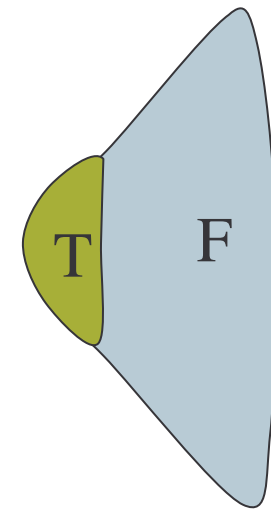
\mathbb{R}^4 topology, one pole at $t = 0$

$\phi \nearrow \phi_f$ as $t \nearrow \infty \implies$ EAdS ($V_F < 0$) or Flat ($V_F = 0$)

$\dot{\rho} > 0 \implies \dot{E} < 0$

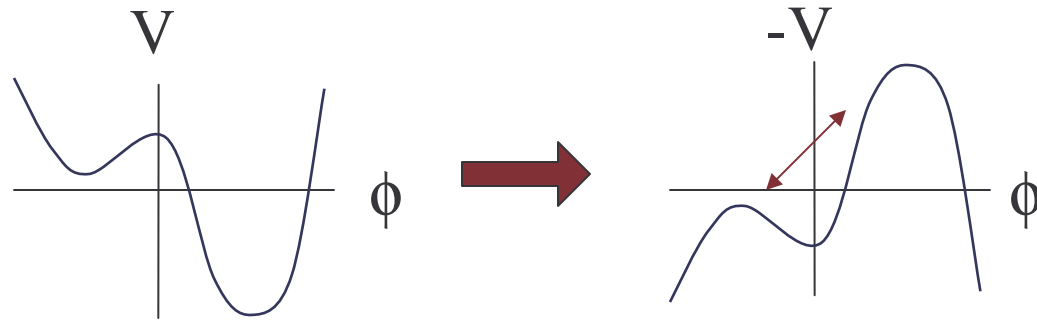
May not reach $\phi_f \implies$ False vac. stable

$S_{\text{BG}} \nearrow \infty \implies$ Need $S_I \nearrow \infty$ for $\Gamma > 0$

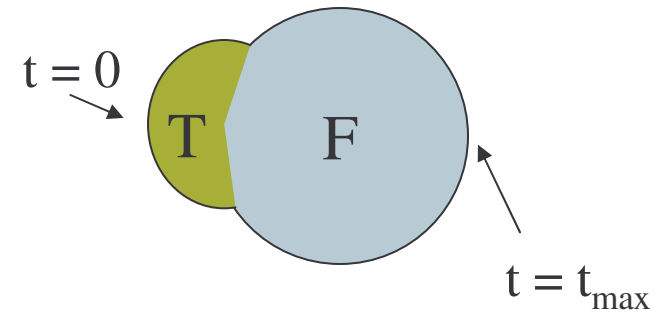


Solutions - Compact

$V_f > 0$:



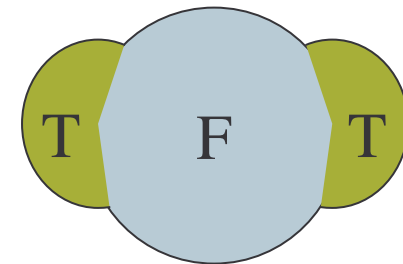
$S^4 \sim \text{EdS}$, two poles at $t = 0, t_{\text{max}}$



$\dot{\rho}$ crosses 0 at equator $\Rightarrow E \uparrow$ (anti-friction)

Always tunneling solution

Multiple passes - $P \geq 0$



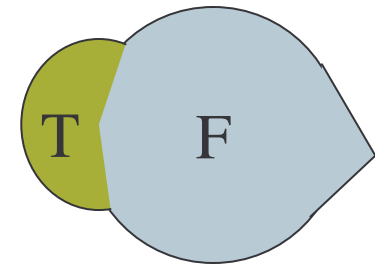
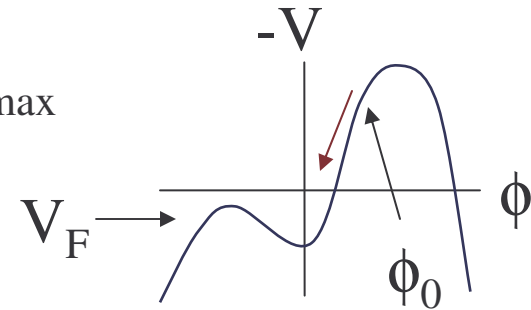
$P=2$

Properties of “Solutions”

“Solution” - solve with (V_F, ϕ_0) \longrightarrow singular **or** regular

- Generically compact with singularity at t_{\max}
- $\phi \nearrow \pm\infty$ for singular “solutions”
- Across reg. compact “sol’n” $\Delta P = 1$

$$\Delta E \begin{cases} > 0 \ni \phi \rightarrow -\infty \\ < 0 \ni \phi \rightarrow \infty, \text{ extra pass} \end{cases}$$

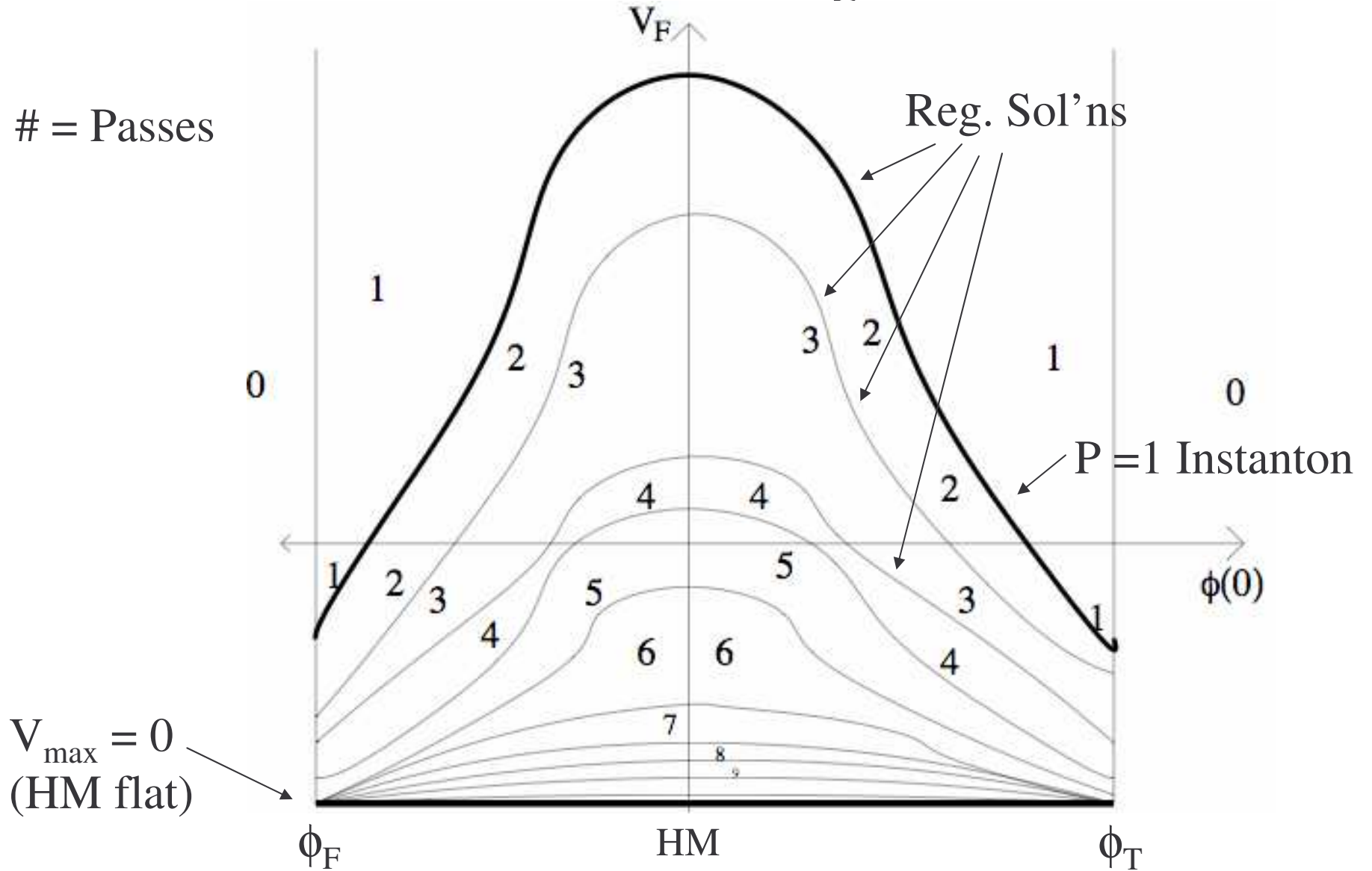


Singular
“solution”

- Across non-compact soln $\Delta P = ?$
- Between ϕ_0^1 and ϕ_0^2 with $\Delta P \neq 0$, reg. sol’n

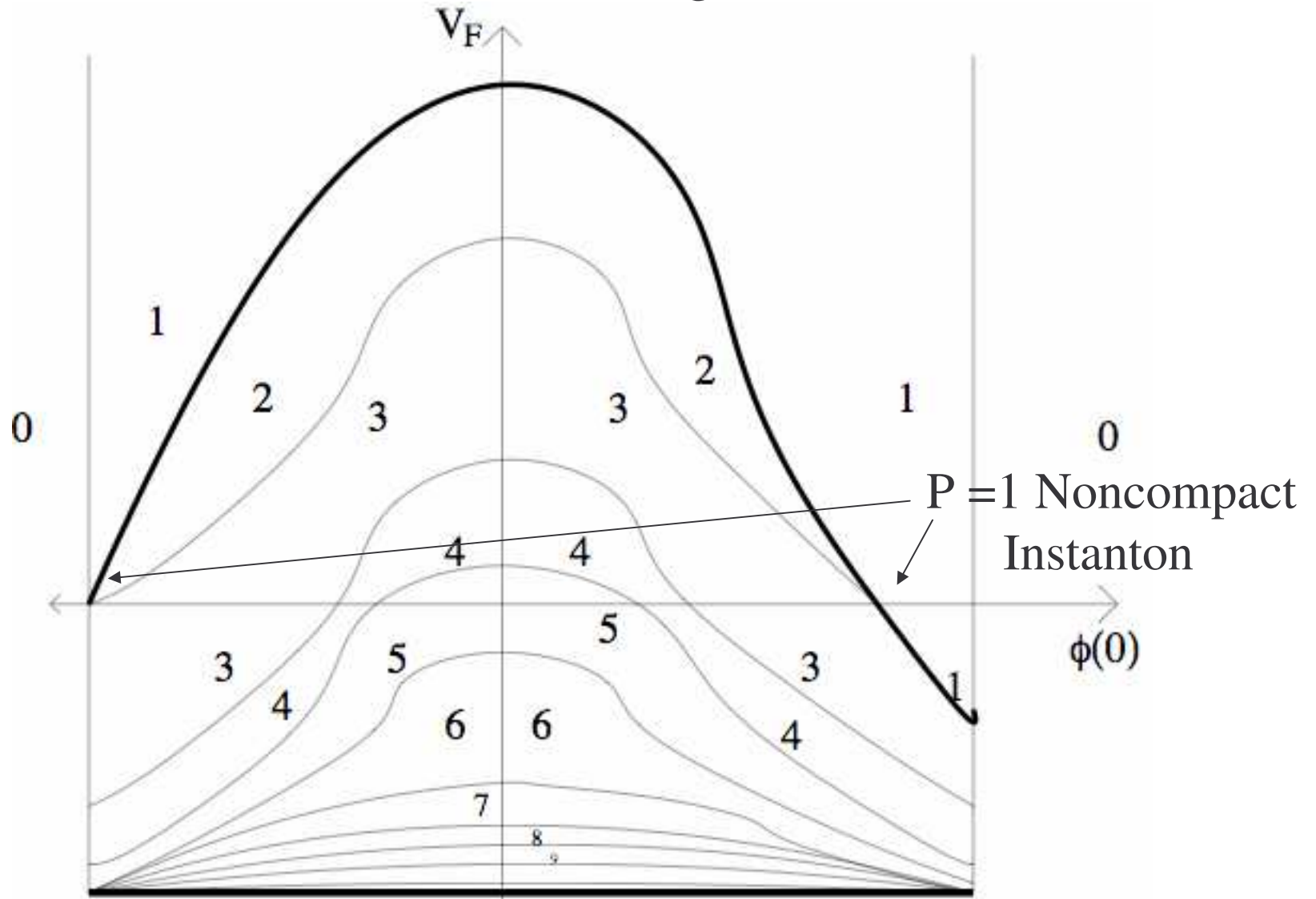
Solution space

No $\Lambda=0$ tunneling



Solution space

$\Lambda=0$ tunneling

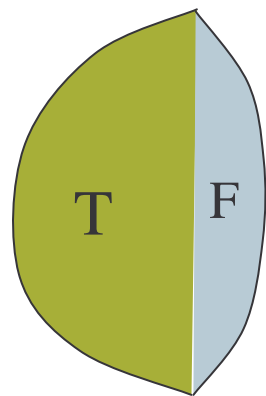


$V_F \not\rightarrow 0$ Limit

Stable $V_F = 0$ False Vacuum

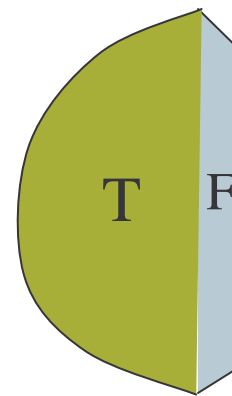
No noncompact solution (by assumption)

Reg. Compact $V_F > 0$ \longrightarrow Reg. Compact $V_F = 0$



Big dS

S_I finite



Flat

S_I finite

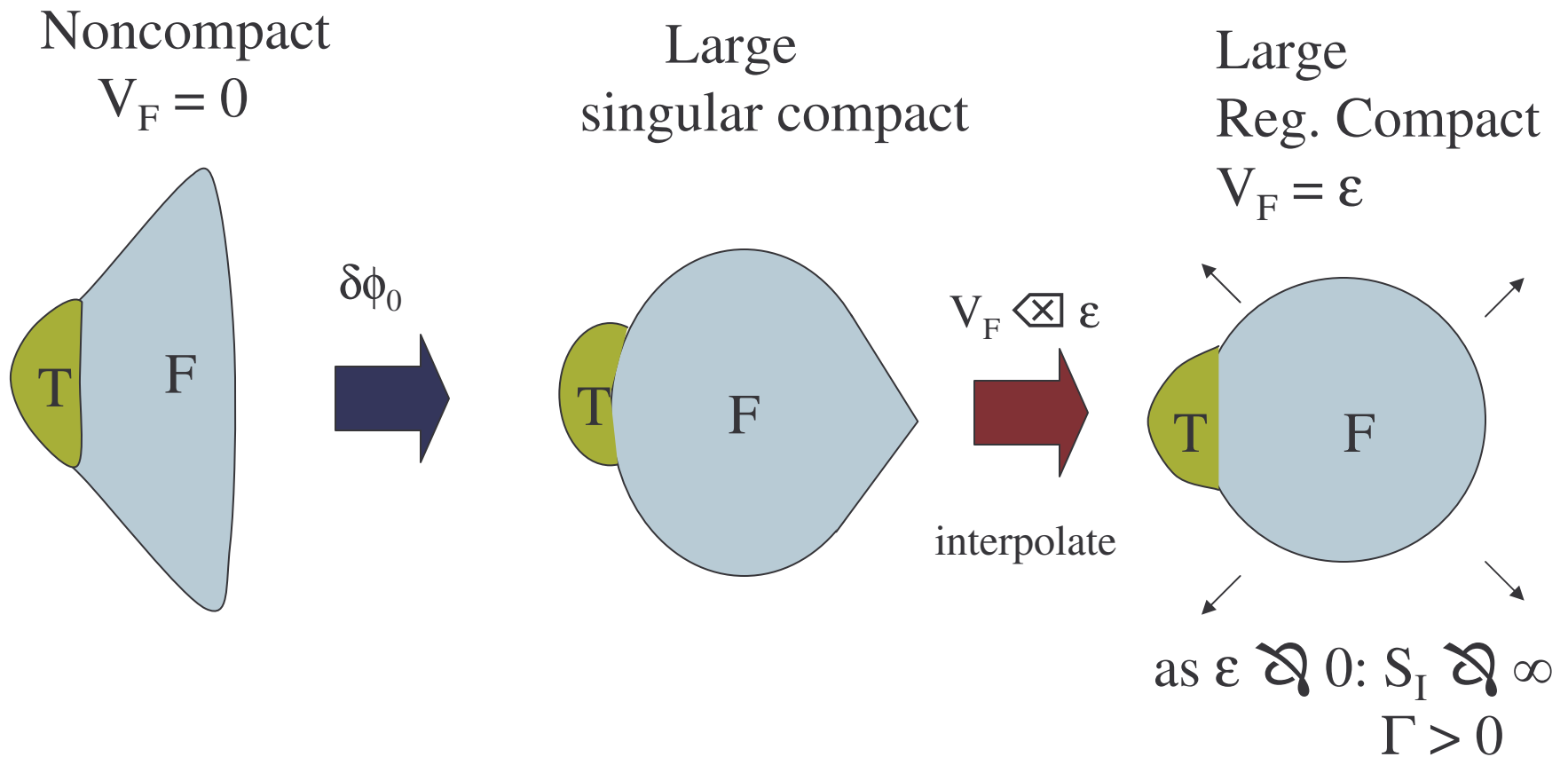
($S_F = \infty$) $\Gamma = 0$

$V_F \not\rightarrow 0$ Limit

Unstable $V_F = 0$ False Vacuum

Noncompact solution exists (by assumption)

Limit discontinuous - hard to perturb



Summary

- Smooth $V_F \rightarrow 0$ limit
 $\Gamma \rightarrow 0 \Rightarrow$ stable flat space
- Ergodic landscape doubtful
- “Solution” space - rich structure