

Black Hole Vacua

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based on work with Andrew,

Introduction

- Andrew enjoyed working on the weird and the geometrical
- The first talk of his I heard was on **pinors**
not a typo!
↓
- Worked on
 - Twistors
 - MM-Theory
- This talk: geometry of black holes

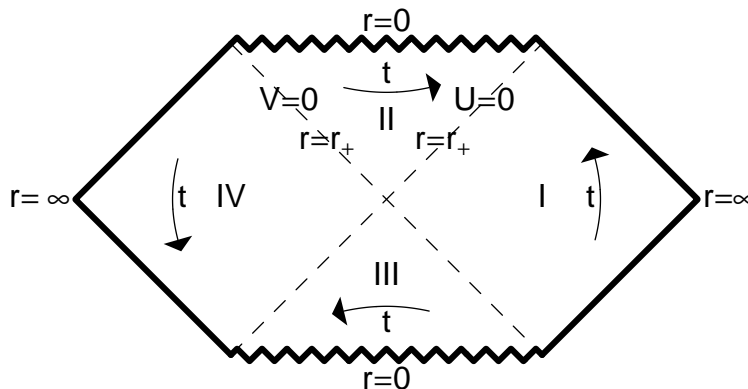
The Schwarzschild Geometry

- Usual (exterior) metric:

$$ds^2 = - \left(1 - \frac{2m}{r} \right) dt^2 + \frac{dr^2}{1 - \frac{2m}{r}} + r^2 d\Omega^2$$

- Kruskal (global) coordinates:

$$ds^2 = e^{-\frac{r}{2m}} \left(16 \frac{m^2}{r} \right) \overbrace{\left(-dT^2 + dZ^2 \right)}^{-2dUdV} + r^2 d\Omega^2$$



- Andrew's Ph.D. advisor studied some discrete symmetries:

- $R_T: (T, Z, \theta, \phi) \rightarrow (-T, Z, \theta, \phi)$ fixed points at $T = 0$
- $R_Z: (T, Z, \theta, \phi) \rightarrow (T, -Z, \theta, \phi)$ fixed points at $Z = 0$
- $P: (T, Z, \theta, \phi) \rightarrow (T, Z, \pi - \theta, \phi + \pi)$ acts freely

Schwarzschild Symmetries

- Of the discrete symmetries R_T, R_Z, P
 - $J = PR_ZR_T$ commutes with all the continuous symmetries of the spacetime
- Has been used to study the time non-orientable quotient Schwarzschild/ J

More Schwarzschild Geometry

We continue à là Andrew and his advisor:

Complexified Schwarzschild is an algebraic variety in \mathbb{C}^7 :

$$\begin{aligned}(Z^6)^2 - (Z^7)^2 + \frac{4}{3}(Z^5)^2 &= 16M^2 \\ \left[(Z^1)^2 + (Z^2)^2 + (Z^3)^2 \right] (Z^5)^4 &= 576M^6 \\ \sqrt{3}Z^4Z^5 + (Z^5)^2 &= 24M^2\end{aligned}$$

Lorentzian section: Z 's real

Euclidean section: $Z^{1\dots 6}$ real, Z^7 imaginary

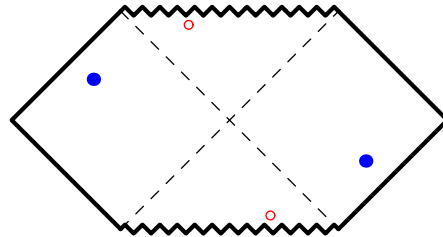
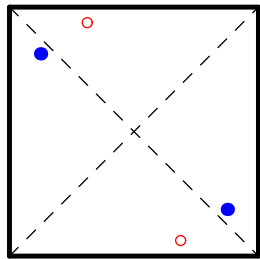
Symmetries:

- $R_Z: Z^6 \rightarrow -Z^6$ rest fixed
- $R_T: Z^7 \rightarrow -Z^7$
- $P: (Z^1, Z^2, Z^3) \rightarrow (-Z^1, -Z^2, -Z^3)$

So what we will say works for Euclidean and Lorentzian black hole.

Black Hole Vacua

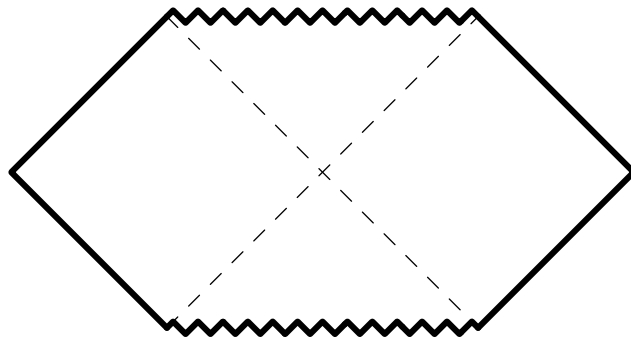
- $J = R_T R_Z P \Rightarrow \mathbb{Z}_2$ “antipodal” map
 - Commutes with all symmetries
- Cf. de Sitter space:



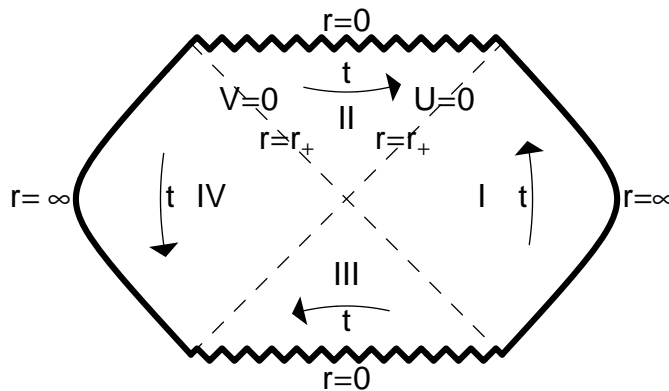
- $\exists \mathbb{Z}_2$ antipodal map $X \rightarrow -X$ on covering space
- Commutes with all symmetries
- Can partially correlate point and antipodal point
 - * \Rightarrow Mottola-Allen transformation
 - * Defines one complex-parameter family of vacua:
 - α -Vacua

Alternative Motivation

- String Theory is supposed to solve quantum gravity problems
 - e.g. Black hole singularity?



- Fidkowski, Hubeny, Kleban and Shenker suggested using AdS/CFT



- Consider geodesics that bounce off singularity
- AdS/CFT relates this to correlators
- Two boundaries \Leftrightarrow **Thermal** F.T.
 - * Time goes in opposite directions on two boundaries
 - * Tr over one boundary \Rightarrow thermal state on other

Generalities

Consider

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-1}^2$$

Where $f(r)$

- monotonically increasing function
- singular at $r = 0$
- zero at $r = r_+ > 0$ [horizon]

Examples:

- Schwarzschild: $f(r) = 1 - \frac{\omega_d M}{r^{d-2}}$
- AdS-Schwarzschild: $f(r) = \frac{r^2}{\ell^2} + 1 - \frac{\omega_d M}{r^{d-2}}$

Fidkowski,
Hubeny,
Kleban,
Shenker

Technicalities:

Define tortoise coordinate:

$$r^* = \int_0^r \frac{dr'}{f(r')} + \frac{\pi i}{f'(r_+)}$$

Double null coordinates:

$$u = t - r^*$$

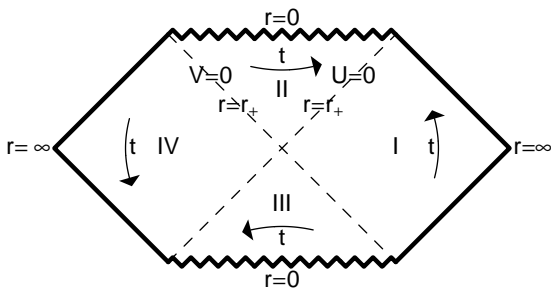
$$v = t + r^*$$

Kruskal coordinates:

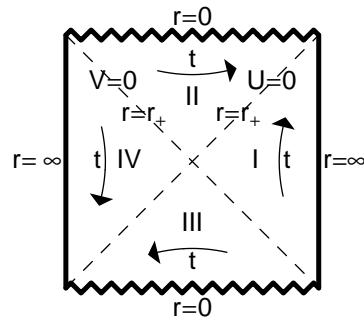
$$U = e^{-\frac{f'(r_+)}{2}u}$$

$$V = e^{\frac{f'(r_+)}{2}v}$$

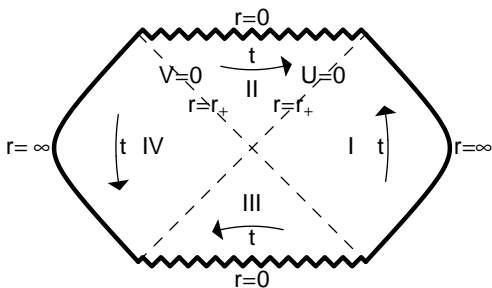
Kruskal coordinates \Leftrightarrow Penrose diagram:



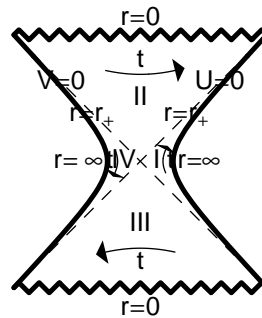
Schwarzschild



BTZ black hole



AdS-Schwarzschild



- Which Penrose \Leftarrow asymptotics of r^*

Symmetries:

- $\frac{\partial}{\partial t}$ is timelike (in asymptotic regions) Killing vector

- S^{d-1} symmetries

$$\frac{1}{2}(U + V)$$

- $T \rightarrow -T$ (vertical reflection of Penrose)

- inverts time (t), preserves S^{d-1}

- fixed points at $T = 0$

$$\frac{1}{2}(-U + V)$$

- $Z \rightarrow -Z$ (horizontal reflection of Penrose)

- inverts time, preserves S^{d-1}

- fixed points at $Z = 0$

- antipodal map of S^{d-1} (not on the Penrose diagram)

- does not affect time; symmetry of S^{d-1}

- no fixed points

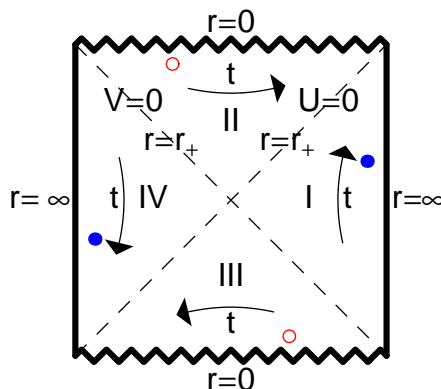
Antipodal Map

Consider combination

$$T \rightarrow -T, Z \rightarrow -Z, \text{ antipodal map on } S^{d-1}$$

- acts freely
- preserves direction of time
- preserves (obvious) symmetries of the space-time
 - commutes with $\frac{\partial}{\partial t}$
 - commutes with S^{d-1} symmetries

Call this the **antipodal map**



Consider a scalar field on this spacetime.

Can choose modes ϕ_n so that

$$\overset{\text{antipodal point}}{\downarrow} \phi_n(x_A) \rightarrow \phi_n(x)^*$$

antipodal map:

positive frequencies \leftrightarrow negative frequencies
(Antipodal map includes time reversal)

Can Bogoliubov standard vacuum:

$$b_n = \cosh \overset{\text{real}}{\downarrow} \alpha a_n - e^{-i\overset{\text{real}}{\downarrow} \gamma} \sinh \alpha a_n^\dagger$$
$$b_n^\dagger = \cosh \alpha a_n^\dagger - e^{i\gamma} \sinh \alpha a_n$$

- One **complex** parameter family of vacua
- Preserve all (obvious) symmetries

No reason to choose one over another ...

α Vacua

These are *exactly* like dS α -vacua

- including construction

But dS α -vacua are frowned upon:

Einhorn,
Larson
Goldstein,
Lowe
Kaloper,
Kleban,
Lawrence,
Shenker,
Susskind

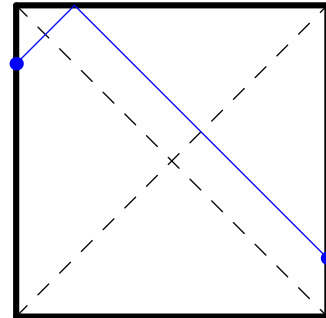
- Causality problems
- Unphysical Poles
- Pinch Singularities
- Not Thermal

We need not have these problems!

Causality

α -vacua

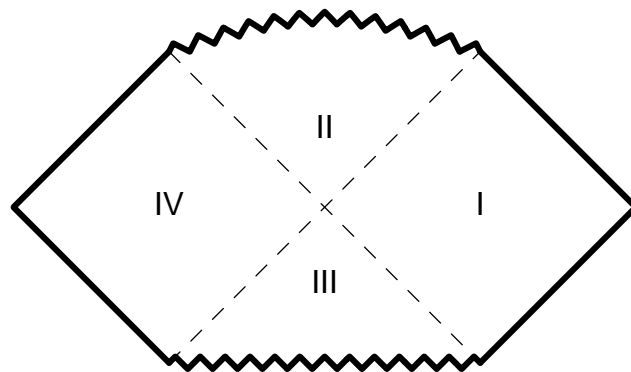
- correlate two *spacelike* points
 - nonintersecting lightcones
- ⇒ No causality problems?



NO!

Leblond,
Marolf,
Myers

- dS gets taller (gravitational backreaction)
- lightcones intersect!



But for **black holes**,

- gravitational backreaction increases horizon size
- lightcones only intersect inside horizon

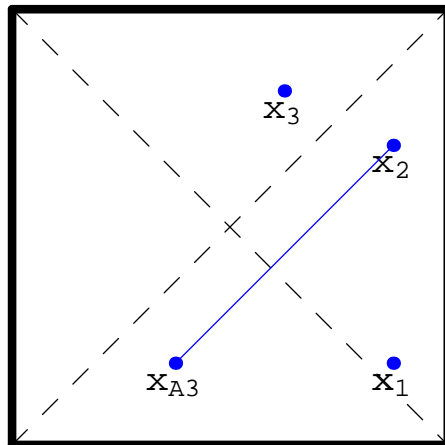
Unphysical Poles

Consider

$$\langle \phi(x_1)\phi(x_2)\phi(x_3) \rangle \sim \int dy G^F(x_1, y) G^F(x_2, y) G^F(x_3, y)$$

For α vacua, poles when y is on lightcone of point *or antipodal point*

- Divergences if, say, also x_2 on lightcone of x_{3A}
 - coincident poles
- Surprising if x_1, x_2, x_3 causally connected



But for **black holes** this requires one of x_1, x_2, x_3 to be inside horizon.

Pinch Singularities

$$\begin{aligned} \text{---} \overset{x}{\circlearrowleft} \text{---} \overset{y}{\circlearrowright} \text{---} &\sim \int dx \int dy G_{\alpha\gamma}^F(x, y) G_{\alpha\gamma}^F(y, x), \\ &\sim \dots + \int dx \int dy \sinh^2 2\alpha G_0^F(x, y) G_0^F(y, x)^* + \dots \end{aligned}$$

Has both $i\epsilon$ prescriptions

\Rightarrow can't evade singularity!

Change time-ordering prescription?

No! Calculate **string** loops (*unlike* dS!)

\Rightarrow No pinch singularities

Thermality

In an α -vacuum

$$\frac{P_{\alpha\gamma}(E_i \rightarrow E_j)}{P_{\alpha\gamma}(E_j \rightarrow E_i)} = \left| \frac{\cosh \alpha + \sinh \alpha e^{i\gamma} e^{\frac{\beta}{2}\Delta E}}{\cosh \alpha + \sinh \alpha e^{i\gamma} e^{-\frac{\beta}{2}\Delta E}} \right|^2 e^{-\beta\Delta E}.$$

- Only thermal (temperature β) if $\alpha = 0$
- Contradicts detailed balance?

$$- \text{i.e. } \rho(E_i)P(E_i \rightarrow E_j) \neq \rho(E_j)P(E_j \rightarrow E_i)$$

NO! Just means *nonequilibrium, steady state*.

Holography?

- For AdS-Schwarzschild, have AdS/CFT.
- Two boundaries \Rightarrow two CFTs
- Ordinary vacuum \Leftrightarrow Pure state of (doubled) CFT
 - Trace over CFT₁ \Rightarrow thermal state of CFT

For α -vacuum, Bogoliubov CFT:

$$\begin{aligned}b_1^\dagger &= \cosh \alpha a_1^\dagger - e^{i\gamma} \sinh \alpha a_2, \\b_2^\dagger &= \cosh \alpha a_2^\dagger - e^{i\gamma} \sinh \alpha a_1\end{aligned}$$

Note “1” and “2” no longer bdy₁ and bdy₂.

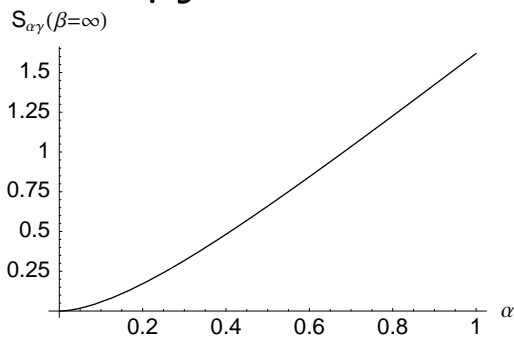
Tr₂ \Rightarrow *nonthermal density matrix*

Agrees with nonthermal formula on AdS side!

Sim. propagators \leftrightarrow correlation functions

Conclusions

- Black holes have α -Vacua
 - Very general
 - For AdS-Schwarzschild, can use CFT as well
 - Entropy at low temperature:



- Entropy at high temperature:

