Black Hole Vacua

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based on work with Andrew,

Introduction

- Andrew enjoyed working on the weird and the geometrical

- The first talk of his I heard was on **p**inors (not a typo!)

- Worked on

  - Twistors

  - MM-Theory

- This talk: geometry of black holes

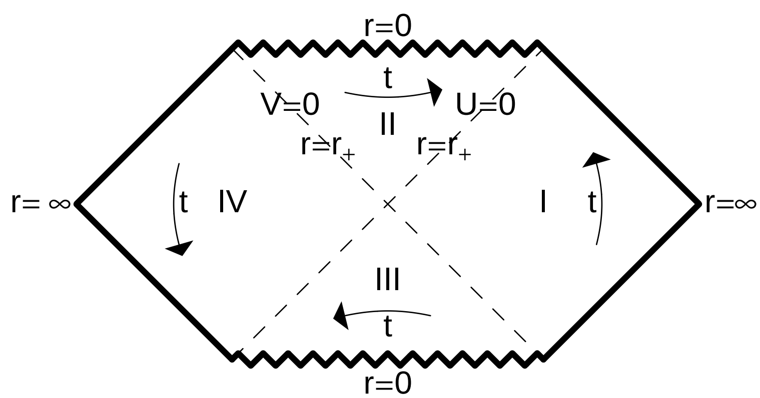
The Schwarzschild Geometry

- Usual (exterior) metric:

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2m}{r}} + r^2 d\Omega^2$$

- Kruskal (global) coordinates:

$$ds^2 = e^{-\frac{r}{2m}} \left(16\frac{m^2}{r}\right) \left(\overbrace{-dT^2 + dZ^2}^{-2dU\,dV}\right) + r^2 d\Omega^2$$



- Andrew's Ph.D. advisor studied some discrete symmetries:

  – R_T: $(T, Z, \theta, \phi) \to (-T, Z, \theta, \phi)$ fixed points at $T = 0$
  – R_Z: $(T, Z, \theta, \phi) \to (T, -Z, \theta, \phi)$ fixed points at $Z = 0$
  – P: $(T, Z, \theta, \phi) \to (T, Z, \pi{-}\theta, \phi{+}\pi)$ acts freely

Schwarzschild Symmetries

- Of the discrete symmetries R_T, R_Z, P
  - $J = PR_ZR_T$ commutes with all the continuous symmetries of the spacetime
- Has been used to study the time non-orientable quotient Schwarzschild/J

More Schwarzschild Geometry

We continue à là Andrew and his advisor:

Complexified Schwarzschild is an algebraic variety in $\mathbb{C}^7$:

$$(Z^6)^2 - (Z^7)^2 + \frac{4}{3}(Z^5)^2 = 16M^2$$
$$\left[(Z^1)^2 + (Z^2)^2 + (Z^3)^2\right](Z^5)^4 = 576M^6$$
$$\sqrt{3}Z^4 Z^5 + (Z^5)^2 = 24M^2$$

Lorentzian section: Z's real
Euclidean section: $Z^{1\ldots6}$ real, Z^7 imaginary

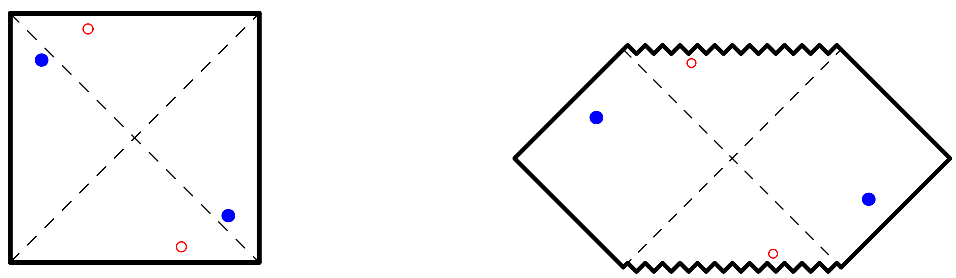
Symmetries:

- R_Z: $Z^6 \to -Z^6$ rest fixed
- R_T: $Z^7 \to -Z^7$
- P: $(Z^1, Z^2, Z^3) \to (-Z^1, -Z^2, -Z^3)$

So what we will say works for Euclidean and Lorentzian black hole.

Black Hole Vacua

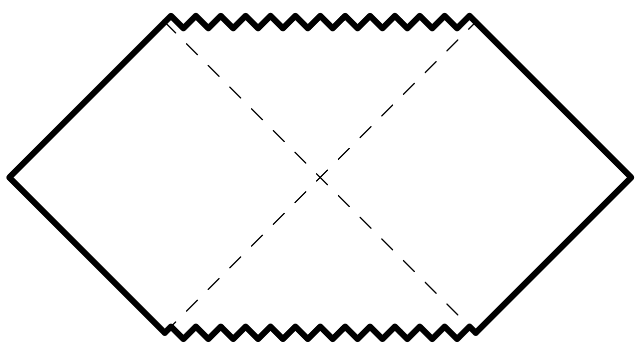
- $J = R_T R_Z P \Rightarrow \mathbb{Z}_2$ "antipodal" map
  - Commutes with all symmetries
- Cf. de Sitter space:



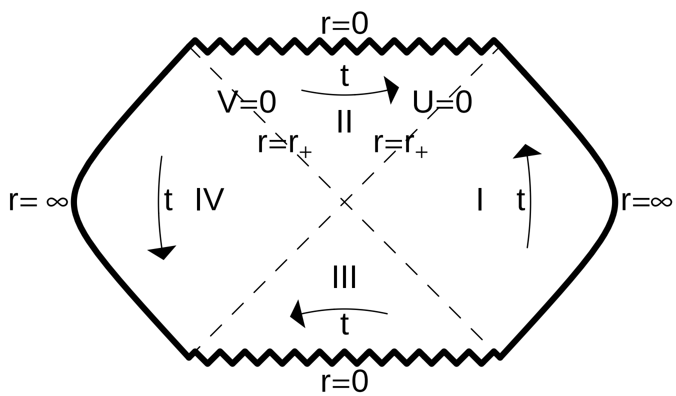
  - $\exists\ \mathbb{Z}_2$ antipodal map $X \to -X$ on covering space
  - Commutes with all symmetries
  - Can partially correlate point and antipodal point
    - $*$ $\Rightarrow$ Mottola-Allen transformation
    - $*$ Defines one complex-parameter family of vacua:
      - · α-Vacua

Alternative Motivation

- String Theory is supposed to solve quantum gravity problems
  - e.g. Black hole singularity?



- Fidkowski, Hubeny, Kleban and Shenker suggested using AdS/CFT



  - Consider geodesics that bounce off singularity
  - AdS/CFT relates this to correlators
  - Two boundaries ⇔ Thermal F.T.
    * Time goes in opposite directions on two boundaries
    * Tr over one boundary ⇒ thermal state on other

Generalities

Fidkowski, Hubeny, Kleban, Shenker

Consider

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-1}^2$$

Where $f(r)$

- monotonically increasing function
- singular at $r = 0$
- zero at $r = r_+ > 0$ [horizon]

Examples:

- Schwarzschild: $f(r) = 1 - \frac{\omega_d M}{r^{d-2}}$
- AdS-Schwarzschild: $f(r) = \frac{r^2}{\ell^2} + 1 - \frac{\omega_d M}{r^{d-2}}$

Technicalities:

Define tortoise coordinate:

$$r^* = \int_0^r \frac{dr'}{f(r')} + \frac{\pi i}{f'(r_+)}$$

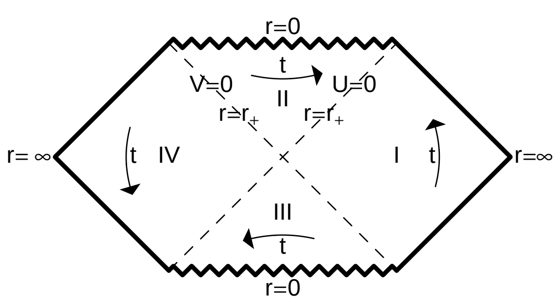
Double null coordinates:

$$u = t - r^* \qquad\qquad v = t + r^*$$

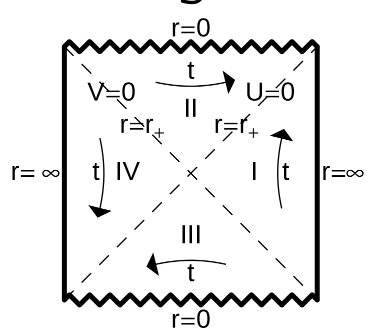
Kruskal coordinates:

$$U = e^{-\frac{f'(r_+)}{2}u} \qquad\qquad V = e^{\frac{f'(r_+)}{2}v}$$

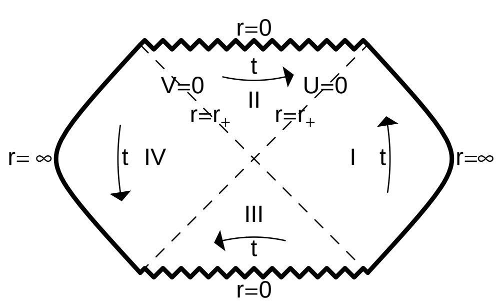
Kruskal coordinates $\Leftrightarrow$ Penrose diagram:



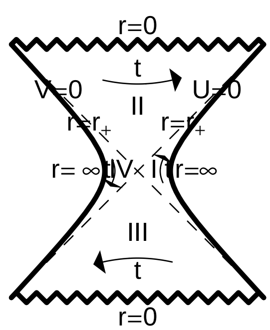
Schwarzschild



BTZ black hole



AdS-Schwarzschild



- Which Penrose $\Leftarrow$ asymptotics of r^*

Symmetries:

- $\frac{\partial}{\partial t}$ is timelike (in asymptotic regions) Killing vector
- S^{d-1} symmetries
- $\overset{\frac{1}{2}(U+V)}{\overset{\downarrow}{T}} \to -T$ (vertical reflection of Penrose)
  - inverts time (t), preserves S^{d-1}
  - fixed points at $T = 0$
- $\overset{\frac{1}{2}(-U+V)}{\overset{\downarrow}{Z}} \to -Z$ (horizontal reflection of Penrose)
  - inverts time, preserves S^{d-1}
  - fixed points at $Z = 0$
- antipodal map of S^{d-1} (not on the Penrose diagram)
  - does not affect time; symmetry of S^{d-1}
  - no fixed points

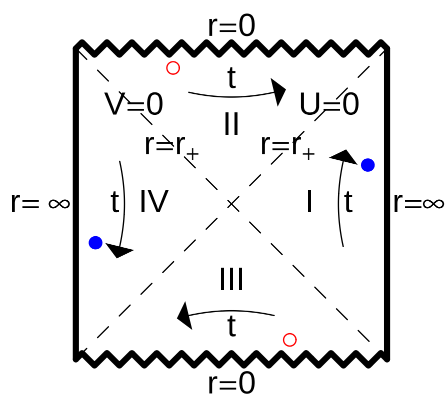
Antipodal Map

Consider combination

$$T \to -T, Z \to -Z, \text{antipodal map on } S^{d-1}$$

- acts freely
- preserves direction of time
- preserves (obvious) symmetries of the space-time
  - commutes with $\frac{\partial}{\partial t}$
  - commutes with S^{d-1} symmetries

Call this the **antipodal** map



Consider a scalar field on this spacetime.

Can choose modes ϕ_n so that

$$\phi_n(\underset{\text{antipodal point}}{x_A}) \to \phi_n(x)^*$$

antipodal map:

positive frequencies $\leftrightarrow$ negative frequencies

(Antipodal map includes time reversal)

Can Bogoliubov standard vacuum:

$$b_n = \cosh \overset{\text{real}}{\alpha}\, a_n - e^{-i\overset{\text{real}}{\gamma}} \sinh \alpha\, a_n^\dagger$$
$$b_n^\dagger = \cosh \alpha\, a_n^\dagger - e^{i\gamma} \sinh \alpha\, a_n$$

- One **complex** parameter family of vacua
- Preserve all (obvious) symmetries

No reason to choose one over another ...

α Vacua

These are *exactly* like dS α-vacua

- including construction

But dS α-vacua are frowned upon:

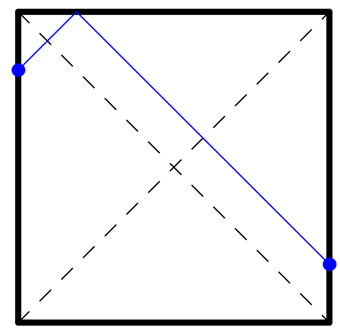
- Causality problems
- Unphysical Poles
- Pinch Singularities
- Not Thermal

Einhorn, Larson
Goldstein, Lowe
Kaloper, Kleban, Lawrence, Shenker, Susskind

We need not have these problems!

Causality

α-vacua



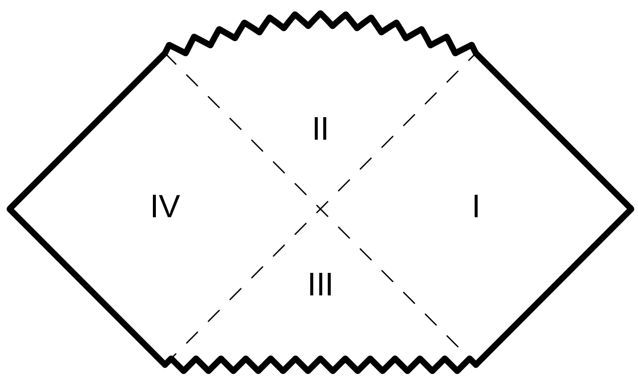
- correlate two *spacelike* points
- nonintersecting lightcones

  $\Rightarrow$ No causality problems?

NO!

Leblond, Marolf, Myers

- dS gets taller (gravitational backreaction)
- lightcones intersect!



But for black holes,

- gravitational backreaction increases horizon size
- lightcones only intersect inside horizon

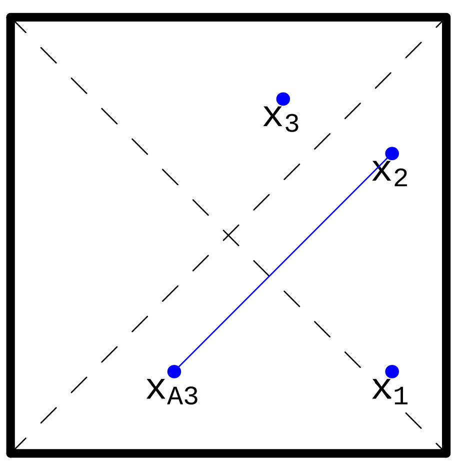
Unphysical Poles

Consider

$$\langle\phi(x_1)\phi(x_2)\phi(x_3)\rangle \sim \int dy G^F(x_1,y)G^F(x_2,y)G^F(x_3,y)$$

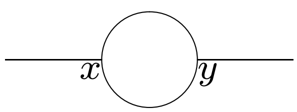
For α vacua, poles when y is on lightcone of point *or antipodal point*

- Divergences if, say, also x_2 on lightcone of x_{3A}
  - coincident poles
- Surprising if x_1, x_2, x_3 causally connected



But for black holes this requires one of x_1, x_2, x_3 to be inside horizon.

Pinch Singularities



$$\sim \int dx \int dy\, G^F_{\alpha\gamma}(x,y) G^F_{\alpha\gamma}(y,x),$$

$$\sim \ldots + \int dx \int dy\, \sinh^2 2\alpha\, G^F_0(x,y) G^F_0(y,x)^* + \ldots$$

Has both $i\epsilon$ prescriptions
  $\Rightarrow$ can't evade singularity!

Change time-ordering prescription?

No! Calculate string loops (*unlike* dS!)
  $\Rightarrow$ No pinch singularities

Thermality

In an α-vacuum

$$\frac{P_{\alpha\gamma}(E_i \to E_j)}{P_{\alpha\gamma}(E_j \to E_i)} = \left| \frac{\cosh\alpha + \sinh\alpha e^{i\gamma} e^{\frac{\beta}{2}\Delta E}}{\cosh\alpha + \sinh\alpha e^{i\gamma} e^{-\frac{\beta}{2}\Delta E}} \right|^2 e^{-\beta\Delta E}.$$

- Only thermal (temperature β) if $\alpha = 0$
- Contradicts detailed balance?
  - i.e. $\rho(E_i)P(E_i \to E_j) \neq \rho(E_j)P(E_j \to E_i)$

NO! Just means *nonequilibrium, steady state*.

Holography?

- For AdS-Schwarzschild, have AdS/CFT.
- Two boundaries ⇒ two CFTs
- Ordinary vacuum ⇔ Pure state of (doubled) CFT
  - Trace over CFT_1 ⇒ thermal state of CFT

For α-vacuum, Bogoliubov CFT:

$$b_1^\dagger = \cosh\alpha\, a_1^\dagger - e^{i\gamma}\sinh\alpha\, a_2,$$
$$b_2^\dagger = \cosh\alpha\, a_2^\dagger - e^{i\gamma}\sinh\alpha\, a_1$$

Note "1" and "2" no longer bdy_1 and bdy_2.

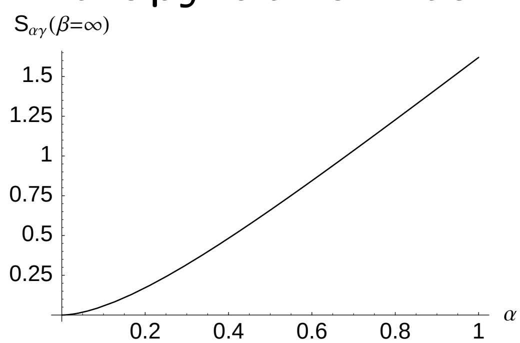
Tr_2 ⇒ *nonthermal* density matrix

Agrees with nonthermal formula on AdS side!

Sim. propagators ↔ correlation functions

Conclusions

- Black holes have α-Vacua
  - Very general
  - For AdS-Schwarzschild, can use CFT as well
  - Entropy at low temperature:



  - Entropy at high temperature: